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Probability Distribution Forecasts: Learning with Random Forests and Graphical Assessment

Moritz N. Lang, Reto Stauffer, Lisa Schlosser, Achim Zeileis

https://topmodels.R-Forge.R-project.org/

Motivation

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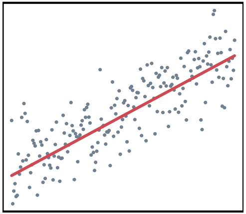
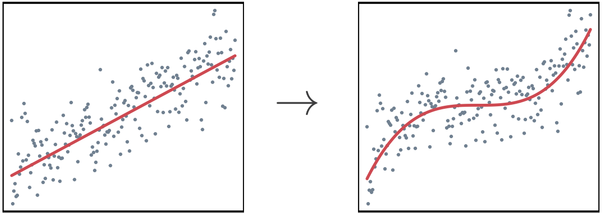


LM, GLM

`lm`
`glm`

Motivation



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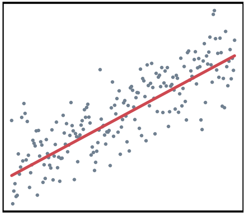
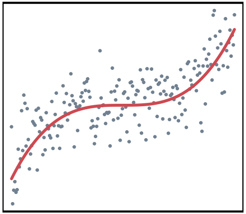
`mgcv`
`VGAM`

Motivation



→



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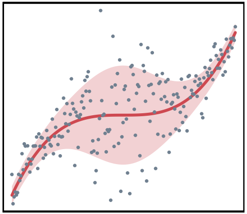

LM, GLM

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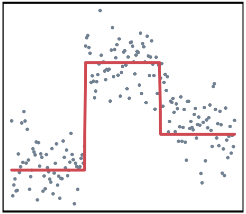
`VGAM`

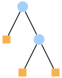
GAMLSS

`gamlss`
`mgcv`
`VGAM`
`gamboostLSS`
`bamlss`

Motivation

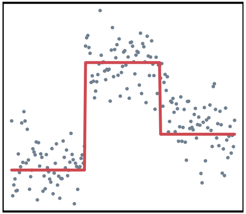


Regression tree



`rpart`
`party(kit)`

Motivation



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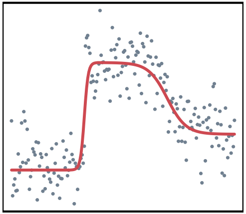

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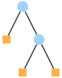

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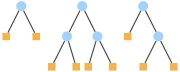
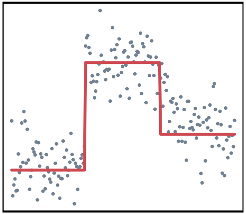


`randomForest`
`ranger`
`party(kit)`

Motivation



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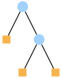

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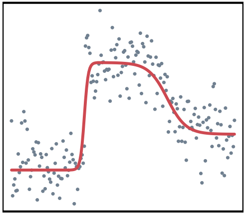

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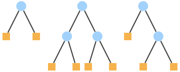

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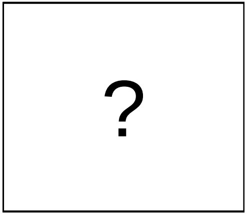

Distributional trees and forests

`disttree`
based on `partykit`

Motivation

Distributional:

- Specify the complete probability distribution (location, scale, shape, ...).

Tree:

- Automatic detection of steps and abrupt changes.
- Capture non-linear and non-additive effects and interactions.

Forest:

- Smoother effects.
- Stabilization and regularization of the model.

Learning distributional trees and forests

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1. Fit global distributional model $\mathcal{D}(Y; \theta)$:
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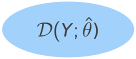

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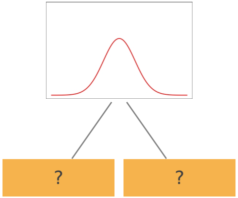

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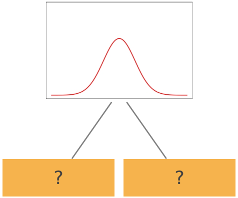

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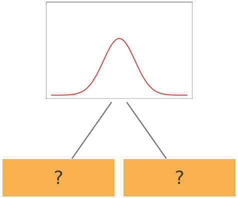

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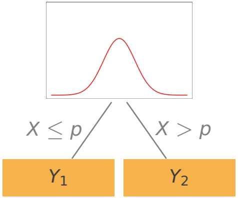

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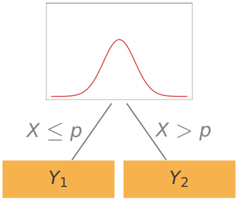

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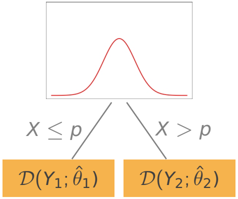

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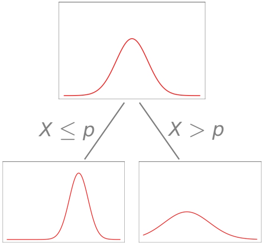

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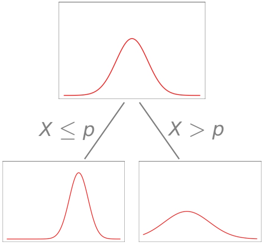

Forest: Ensemble of T trees.

- Bootstrap or subsamples.
- Random input variable sampling.

Application

Goal: Probabilistic precipitation forecasting.

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Covariates:

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Distribution assumption: Power-transformed Gaussian, censored at 0.

$$(\text{precipitation})^{\frac{1}{1.6}} \sim c\mathcal{N}(\mu, \sigma^2)$$

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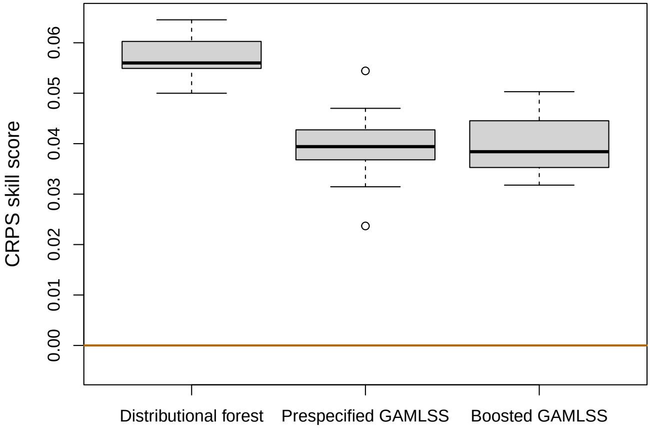
Application

Predictive performance: Distributional forests improve CRPS skill score compared to heteroscedastic linear model (EMOS) and competing GAMLSS.



Graphical assessment

However: Is the distributional fit calibrated?

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Graphical assessments: Various possibilities suggested in different parts of the literature.

- (Randomized) quantile-quantile residuals plot.
- Probability integral transform (PIT) histogram.
- Rootogram.
- Reliability diagram at prespecified thresholds.
- Worm plot.

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Now: `topmodels` (on R-Forge).

Graphical assessment

Packages and data:

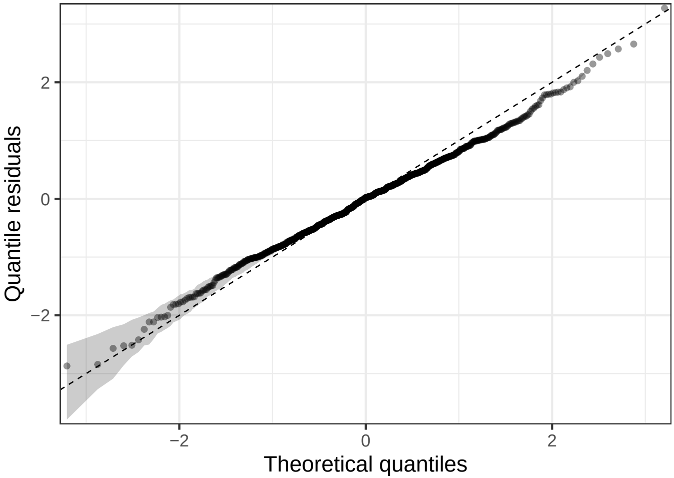
```
R> install.packages("disttree", repos = "https://R-Forge.R-project.org")
R> install.packages("topmodels", repos = "https://R-Forge.R-project.org")
R> library("disttree")
R> library("topmodels")
R> data("RainAxams", package = "disttree")
```

Random forest:

```
R> forest <- distforest(robs ~ .,
+                       family = dist_list_cens_normal,
+                       data = RainAxams, ...)
```

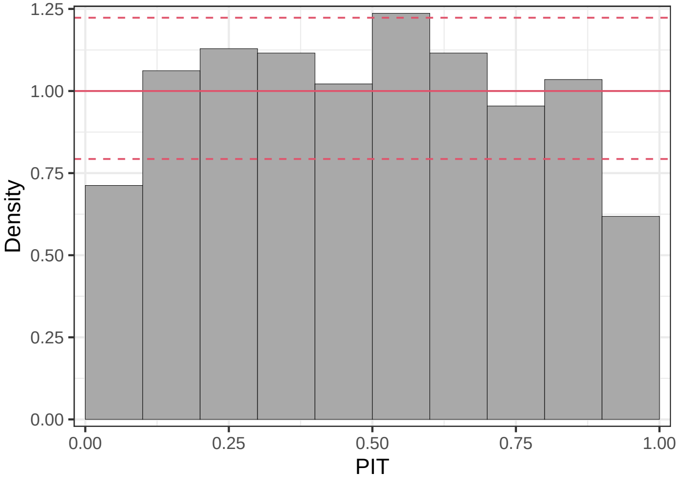
Graphical assessment

Q-Q residuals plot: `qqrplot(forest)`



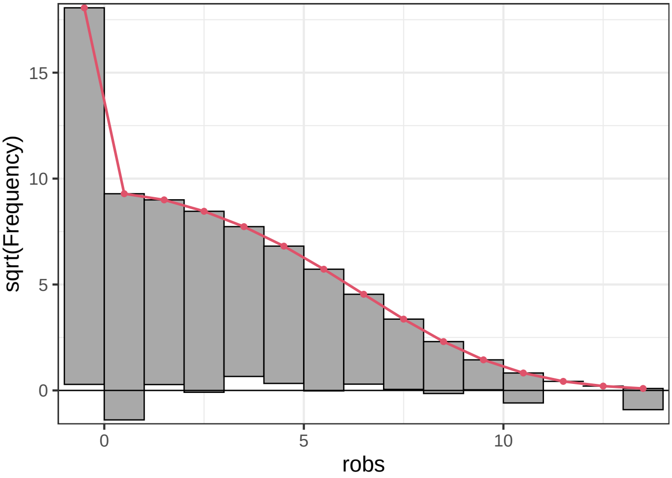
Graphical assessment

PIT histogram: `pithist(forest)`



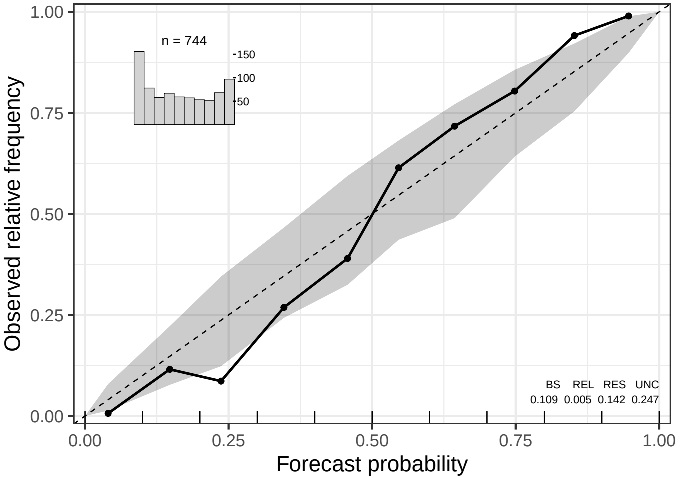
Graphical assessment

Rootogram: `rootogram(forest)`



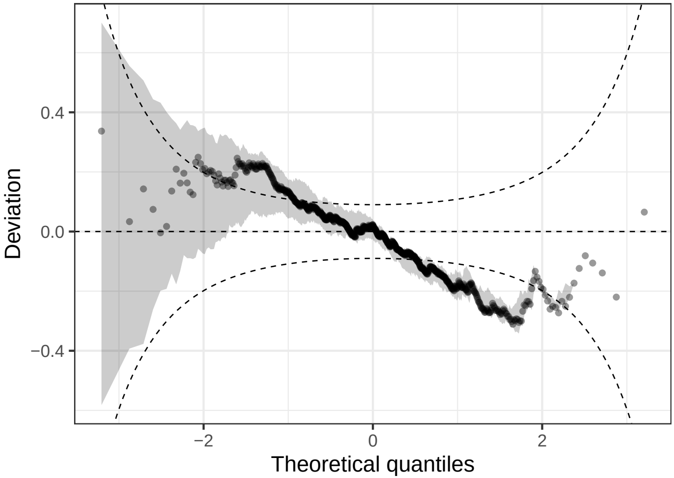
Graphical assessment

Reliability diagram: `reliagram(forest)`



Graphical assessment

Worm plot: `wormplot(forest)`



Graphical assessment

In contrast: Linear Gaussian model.

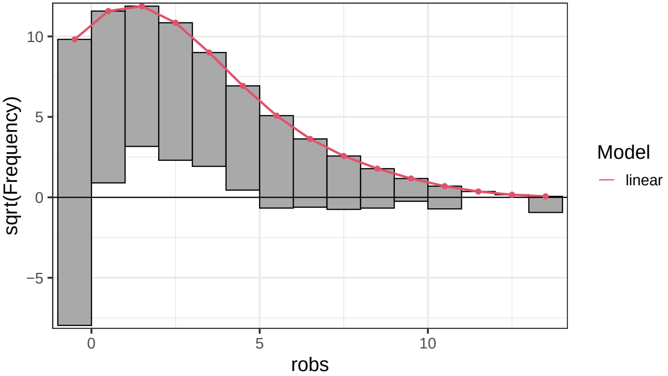
- Homoscedastic.
- Not accounting for excess zeros.
- Incorrect assumption of underlying response distribution.

```
R> linear <- lm(robs ~ tppow_mean, data = RainAxams)
```

Graphical assessment

Model comparison: Rootogram

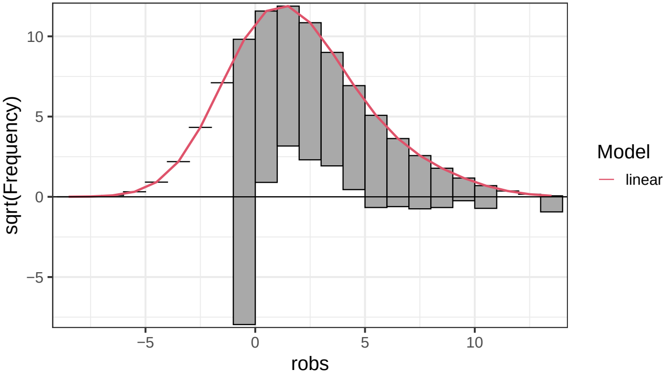
```
R> rootogram(linear, plot = FALSE) |>
+    autoplot(legend = TRUE)
```



Graphical assessment

Model comparison: Rootogram

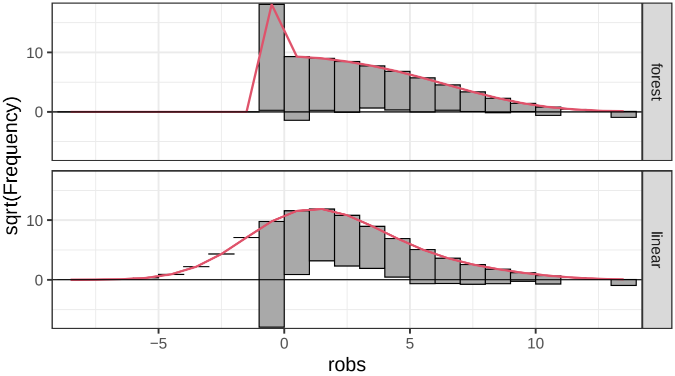
```
R> rootogram(linear, plot = FALSE, breaks = -9:14) |>
+    autoplot(legend = TRUE)
```



Graphical assessment

Model comparison: Rootogram

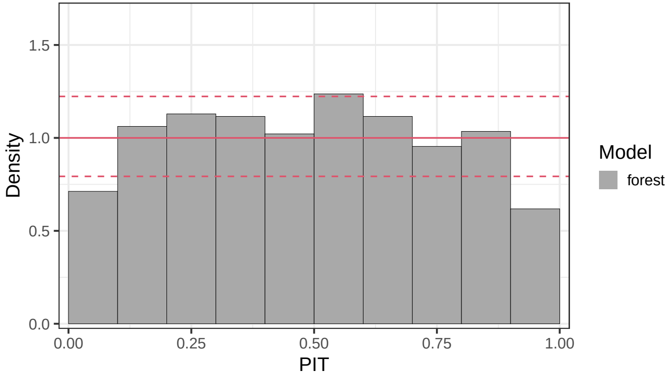
```
R> c(rootogram(forest, breaks = -9:14), rootogram(linear, breaks = -9:14)) |>
+    autoplot(legend = TRUE)
```



Graphical assessment

Model comparison: PIT histogram

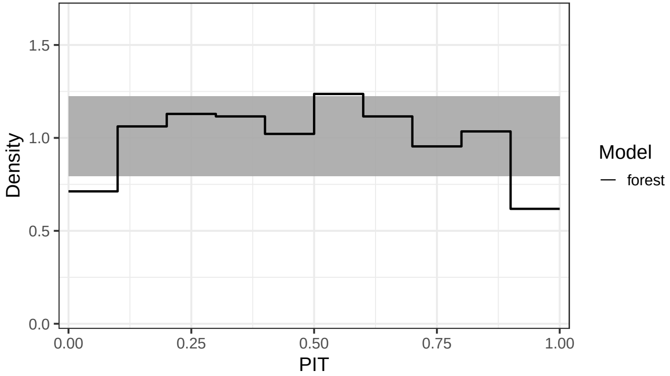
```
R> pithist(forest, plot = FALSE) |>
+    autoplot(legend = TRUE)
```



Graphical assessment

Model comparison: PIT histogram

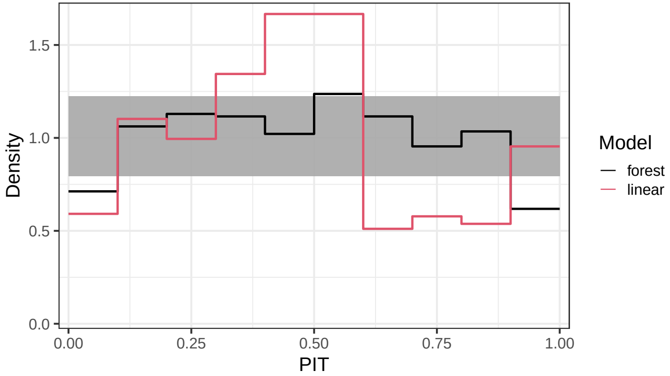
```
R> pithist(forest, plot = FALSE) |>
+    autoplot(legend = TRUE, style = "lines")
```



Graphical assessment

Model comparison: PIT histogram

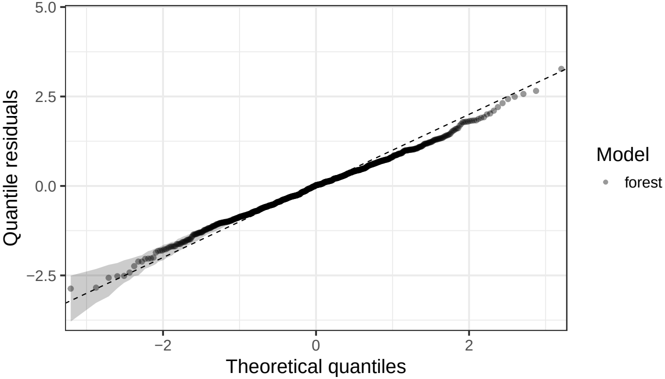
```
R> c(pithist(forest, plot = FALSE), pithist(linear, plot = FALSE)) |>
+    autoplot(legend = TRUE, style = "lines", single_graph = TRUE, col = 1:2)
```



Graphical assessment

Model comparison: Q-Q residuals plot

```
R> qqrplot(forest, plot = FALSE) |>
+     autoplot(legend = TRUE)
```



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Model comparison: Q-Q residuals plot

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R> c(qqrplot(forest, plot = FALSE), qqrplot(linear, plot = FALSE)) |>
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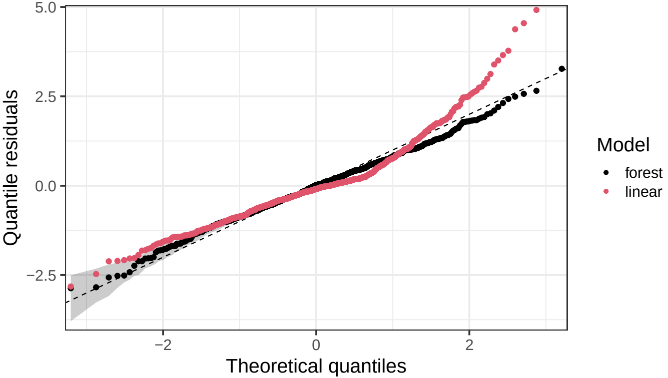

Software

disttree: available on R-Forge at

`https://R-Forge.R-project.org/projects/partykit/pkg/disttree/`

Concept: Fusion of tree-based models with distributional modeling.

Main functions:

`distfit`	Distributional fits (ML, `gamlss.family`/custom `list`). No covariates.
`disttree`	Distributional trees (`ctree`/`mob` + `distfit`). Covariates as partitioning variables.
`distforest`	Distributional forests (ensemble of `disttrees`). Covariates as partitioning variables.

Software

topmodels: available on R-Forge at

`https://topmodels.R-Forge.R-project.org/`

Concept: Unifying toolbox for probabilistic forecasts and graphical model assessment.

Main functions:

`procast`	Probabilistic forecasts (`(g)lm`, `crch`, `disttree`, more to come).
	Computation of probabilities, densities, scores, and Hessians.
`rootogram`, `pithist`, ...	Plotting rootograms, PIT histograms, ...
`plot, autoplot`	Generic `plot`, `autoplot` function.

References

Schlosser L, Hothorn T, Stauffer R, Zeileis A (2019). "Distributional Regression Forests for Probabilistic Precipitation Forecasting in Complex Terrain." *The Annals of Applied Statistics*, **13**(3), 1564–1589. doi:10.1214/19-AOAS1247

Lang MN, Zeileis A *et al.* (2021). "topmodels: Infrastructure for Inference and Forecasting in Probabilistic Models." *R package version 0.1-0*. https://topmodels.R-Forge.R-project.org/

Hothorn T, Zeileis A (2015). "partykit: A Modular Toolkit for Recursive Partytioning in R." *Journal of Machine Learning Research*, **16**, 3905–3909. http://www.jmlr.org/papers/v16/hothorn15a

https://topmodels.R-Forge.R-project.org/

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